"To accomplish anything of significance comes with a cost. Are you willing to pay the price?"

Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Topic</th>
<th>Page #s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Inequalities</td>
<td>1 – 3</td>
</tr>
<tr>
<td>2.</td>
<td>Exponents and Radicals</td>
<td>4 – 7</td>
</tr>
<tr>
<td>3.</td>
<td>Algebraic Expressions</td>
<td>8 – 11</td>
</tr>
<tr>
<td>4.</td>
<td>Rational Expressions</td>
<td>12 – 15</td>
</tr>
<tr>
<td>5.</td>
<td>Solving Equations</td>
<td>16 – 19</td>
</tr>
<tr>
<td>6.</td>
<td>Answers</td>
<td>20</td>
</tr>
</tbody>
</table>

Summer Reading

Each student taking Regular Pre-Calculus or Pre-Calculus AB next fall at Bethlehem Central High School is responsible for reading this material and completing all of the practice problems at the end of each section. Each set of problems are to be completed on a separate sheet of paper (blank electronic sheet), showing all work. All student work should be submitted electronically via google classroom, by the first day of class. You will receive your schedule and assigned pre-calculus teacher by the end of August. If this is not possible bring the completed work to the first day of class. Although these concepts will not be directly assessed they lay the foundation for the topics we study in the first semester of the year.
1. **Inequalities**

**Inequalities are statements that two quantities are not equal.**

1) To solve the inequality means to find all the solutions to the statement.
2) Most inequalities have an infinite number of solutions.
3) Solving an inequality is similar to solving an equation.
4) Both sides of the inequality can be added to or subtracted from.
5) Multiplying or dividing an inequality by a negative number reverses the inequality sign.

6) **Example:** solve for $x$: 
   
   \[
   \begin{align*}
   5x - 2 & \leq -2x + 12 \\
   7x - 2 & \leq 12 \\
   7x & \leq 14 \\
   x & \leq 2 
   \end{align*}
   \]

7) **Notation:**

   \[\begin{array}{c}
   0 \quad 1 \quad 2 \\
   \text{or} \\
   0 \quad 1 \quad 2
   \end{array}\]

8) **Interval notation:**

   \[(-\infty, 2]\]

9) **Example:** solve

   \[
   \frac{5}{x - 6} \geq 0
   \]

   Since the numerator is positive, the fraction will be positive whenever the denominator is positive:

   therefore \(x - 6 \geq 0\) and \(x \geq 6\) or \([6, \infty)\)

10) **Solving an Absolute Value Inequality**

   If \(X\) is an algebraic expression and \(c\) is a positive number,

   a. The solutions of \(|X| < c\) are the numbers that satisfy \(-c < X < c\).

   b. The solutions of \(|X| > c\) are the numbers that satisfy \(X < -c\) or \(X > c\).

   These rules are valid if \(<\) is replaced by \(\leq\) and \(>\) is replaced by \(\geq\).

11) **Solve:**

   \[|2x + 3| \geq 5\]

   \[|x| \geq c\] means \(x \leq -c\) or \(x \geq c\)

   \[|2x + 3| \geq 5\] means \(2x + 3 \leq -5\) or \(2x + 3 \geq 5\)

   We solve each of these inequalities separately.

   \[
   \begin{align*}
   2x + 3 & \leq -5 \\
   2x & \leq -8 \\
   x & \leq -4 \\
   \text{or} \\
   2x + 3 & \geq 5 \\
   2x & \geq 2 \\
   x & \geq 1
   \end{align*}
   \]

   The solution set is \(\{x | x \leq -4 \text{ or } x \geq 1\}\),

   that is, all \(x\) in \((-\infty, -4]\) or \([1, \infty)\).
Problem Set #1:

Solve the inequalities. Express answer in interval notation.

1) \(3x + 2 < 5x - 9\)

2) \(1 \leq \frac{2x + 1}{3} \leq 5\)

3) \(-1 \leq 3 - 2x < 5\)

4) \(\frac{4x - 3}{6} + 2 \geq \frac{2x - 1}{12}\)

5) \(|2x - 6| < 8\)

6) \(|3x - 8| > 7\)

7) \(|\frac{2y + 6}{3}| < 2\)

8) \(|3 - \frac{3}{4}x| > 9\)

9) \(3|x - 1| + 2 \geq 8\)

10) \(-2|4 - x| \geq -4\)
2. **Exponents and Radicals**

**Integer Exponents**

In the expression $a^n$, $a$ is called the base and $n$ is called the exponent. The base can be anything from a number to a variable or an algebraic expression.

**Laws of Exponents**

1. $a^m a^n = a^{m+n}$
   
   Example: $2^3 \cdot 2^4 = 2^{3+4} = 2^7$

2. $(a^m)^n = a^{mn}$
   
   Example: $(3^3)^2 = 3^{2\times 3} = 3^6$

3. $(ab)^n = a^n b^n$
   
   Example: $(20)^3 = (2 \cdot 10)^3 = 2^3 \cdot 10^3 = 8 \cdot 1000 = 8000$

4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
   
   Example: $\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$

5. $\frac{a^m}{a^n} = a^{m-n}$
   
   Example: $\frac{2^5}{2^7} = 2^{5-7} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

6. $a^{-n} = \frac{1}{a^n}$
   
   Example: $(-3)^{-5} = \frac{1}{(-3)^5} = \frac{1}{-243}$

**Radical Notation**

The expression $\sqrt[n]{a}$ is called the principal $n$th root of $a$. This expression is called a *radical*, the number "$a$" is called the *radicand*, and the number $n$ is called the index of the radical.

If $n = 2$, we do not write the index. The principal $n$th root is defined as follows:

*Let "$n$" be a positive integer greater than 1 and let "$a" be a real number.*

**A. Rules and examples:**

1. if $a = 0$, then $\sqrt[n]{a} = 0$

2. if $a > 0$, then $\sqrt[n]{a}$ is the positive real number $b$ such that $b^n = a$

3. if $a < 0$ and $n$ is odd, then $\sqrt[n]{a}$ is the negative real number $b$ such that $b^n = a$

4. if $a < 0$ and $n$ is even, then $\sqrt[n]{a}$ is not a real number

5. $\sqrt{16} = 4$

6. $\sqrt[3]{32} = 2$

7. $\sqrt[3]{-8} = -2$

8. $\sqrt[4]{-16}$ is not a real number
B. Properties and laws of radicals with positive integer index

1) \((\sqrt[n]{a})^n = a\), if \(\sqrt[n]{a}\) is a real number
   Example: \((-8)^{\frac{3}{2}} = -8\)

2) \(\sqrt[n]{a^n} = a\), if \(a \geq 0\)
   Example: \(\sqrt{5^2} = 5\)

3) \(\sqrt[n]{a^n} = a\), if \(a < 0\) and \(n\) is odd
   Example: \(\sqrt[3]{(-2)^3} = -2\)

4) \(\sqrt[n]{a^n} = |a|\), if \(a < 0\) and \(n\) is even
   Example: \(\sqrt[4]{(-2)^4} = |-2| = 2\)

Note that \(\sqrt[4]{(-2)^4} = 2\), but \((-2)^{\frac{4}{2}}\) is not a real number since \(\sqrt{-2}\) is not a real number. The following laws are true as long as each root is a real number:

1) \(\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}\)

2) \(\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}\)

3) \(\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}\)

C. Simplifying radical expressions

1) To simplify a radical expression with index \(n\), we look for factors of the radicand that can be written with an exponent of \(n\).

   To simplify \(\sqrt[3]{320}\), we look for any factors of 320 that are perfect cubes:
   \[
   \sqrt[3]{320} = \sqrt[3]{64 \cdot 5} = \sqrt[3]{64} \cdot \sqrt[3]{5} = 4\sqrt[3]{5}
   \]

   Another example is:
   \[
   \sqrt[3]{16x^3y^6z^4} = \sqrt[3]{8 \cdot 2 \cdot x^3 \cdot y^6 \cdot z^3 \cdot z} = \sqrt[3]{8x^2y^2z^3} \cdot \sqrt[3]{2y^2z} = 2xy^2z\sqrt[3]{2y^2z}
   \]

2) If the denominator of a fraction contains an \(n\)th root radical, to simplify the radical, we need to rationalize the denominator. This process is done by multiplying the numerator and denominator of the fraction by an \(n\)th root radical to end up with an \(n\)th power in the radicand.

Examples:

   a) \[
   \frac{1}{\sqrt[5]{5}} = \frac{1}{\sqrt[5]{5}} \cdot \sqrt[5]{5} = \frac{\sqrt[5]{5}}{\sqrt[5]{5^2}} = \frac{\sqrt[5]{5}}{5}
   \]
b) \[ \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x^2}}{\sqrt{x^2}} = \frac{\sqrt{x^2}}{x} = \frac{x}{x} \]

3) If an expression has an exponent that is a rational number but is not an integer, we can use radicals to define it.

   a) \( x^{\frac{1}{2}} = \sqrt{x} \)
   b) \( x^{\frac{2}{3}} = \left(\sqrt[3]{x}\right)^2 = \left(\sqrt[3]{x^2}\right) = \sqrt[6]{x^2} \cdot x^\frac{1}{2} = x \sqrt[6]{x^2} \)
   c) \( 125^{\frac{2}{3}} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25 \)
   d) \( x^{\frac{3}{2}} = \sqrt{x^3} = \sqrt{x^2 x} = |x| \sqrt{x} \)

4) Rationalizing a Denominator Containing Two Terms

   Example: Rationalize the denominator: \( \frac{7}{5 + \sqrt{3}} \).

   Solution: If we multiply the numerator and denominator by \( 5 - \sqrt{3} \), the denominator will not contain a radical. (Remember \((\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b\) ) Therefore, we multiply by 1, choosing \( \frac{5 - \sqrt{3}}{5 - \sqrt{3}} \) for 1.

   \[
   \frac{7}{5 + \sqrt{3}} = \frac{7}{5 + \sqrt{3}} \cdot \frac{5 - \sqrt{3}}{5 - \sqrt{3}} = \frac{7(5 - \sqrt{3})}{5^2 - (\sqrt{3})^2} = \frac{7(5 - \sqrt{3})}{25 - 3} = \frac{7(5 - \sqrt{3})}{22} \text{ or } \frac{35 - 7\sqrt{3}}{22}
   \]

   In either form of the answer, there is no radical in the denominator.
Problem Set #2:

Simplify the following, and rationalize the denominator when appropriate. Your answers should have no negative or rational exponents.

1) \((-3x^{-2})(4x^4)\)  
2) \((-3a^2b^{-5})^3\)

3) \(\frac{(3y^3)(2y^2)^2}{(y^4)^3}\)  
4) \(\left(\frac{x^6}{9y^{-4}}\right)^{-\frac{1}{2}}\)

5) \(a^{\frac{3}{2}}a^{-\frac{1}{2}}a^{\frac{1}{6}}\)  
6) \(\sqrt[3]{8a^6b^{-3}}\)

7) \(\sqrt[3]{\frac{3x}{2y^3}}\)  
8) \(\frac{\sqrt[3]{2x^4y^4}}{9x}\)

9) \(\sqrt[2]{432x^5y^2z^6}\)  
10) \(\frac{\sqrt[3]{-8x^6}}{3x^{11}}\)

11) \(\frac{7}{\sqrt{5} - 2}\)  
12) \(\frac{11}{\sqrt{7} - \sqrt{3}}\)
3. **Algebraic Expressions**

A polynomial in \( x \) is a sum of terms in the form:

\[
 a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0
\]

Where \( n \) is a non-negative integer and each coefficient \( a_k \) is a real number, \( a_n \neq 0 \), \( a_n \) is the lead coefficient of the polynomial and \( n \) is the degree of the polynomial.

\[
3x^4 + 5x^3 + (-7x) + 4 \quad \text{has a lead coefficient of 3 and degree 4}
\]

\[
x^8 + 9x^6 - 2 \quad \text{has a lead coefficient of 1 and degree 8}
\]

\[
8 \quad \text{has a lead coefficient of 8 and degree 0}
\]

A. **Polynomials**

1) **Terms**

   a) A polynomial with 1 term is a monomial
   b) A polynomial with 2 terms is a binomial
   c) A polynomial with 3 terms is a trinomial

2) **Degrees**

   a) A polynomial of degree 0 is a constant polynomial
   b) A polynomial of degree 1 is a linear polynomial
   c) A polynomial of degree 2 is a quadratic polynomial
   d) A polynomial of degree 3 is a cubic polynomial

B. **To add or subtract polynomials, add or subtract like terms**

1) \( (2x^2 + 3x - 5) + (-3x^2 + 2x + 4) \)

2) \( = (2x^2 - 3x^2) + (3x + 2x) + (-5 + 4) \)

3) \( = (-x^2 + 5x - 1) \)

C. **To multiply polynomials, use the distributive property**

\[
(x^2 + 2x - 3)(2x^2 - 3x + 1) = x^2(2x^2 - 3x + 1) + 2x(2x^2 - 3x + 1) - 3(2x^2 - 3x + 1) \\
= 2x^4 - 3x^3 + x^2 + 4x^3 - 6x^2 + 2x - 6x^2 + 9x - 3 \\
= 2x^4 + x^3 - 11x^2 + 11x - 3
\]
D. Some special products

1) \((x + y)(x - y) = x^2 - y^2\)
2) \((x + y)^2 = x^2 + 2xy + y^2\)
3) \((x - y)^2 = x^2 - 2xy + y^2\)
4) \((x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3\)
5) \((x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3\)

E. Examples of special products

1) \((2x + 3y)(2x - 3y) = (2x)^2 - (3y)^2 = 4x^2 - 9y^2\)
2) \((a - 5)^2 = a^2 - 2(a)(5) + 5^2 = a^2 - 10a + 25\)
3) \((2x + 4)^3 = (2x)^3 + 3(2x)^2(4) + 3(2x)(4)^2 + 4^3 = 8x^3 + 48x^2 + 96x + 64\)

F. To factor a polynomial means to write it as a product of prime polynomials.

1) Some special factorizations
   a) Difference of two perfect squares \(x^2 - y^2 = (x + y)(x - y)\)
   b) Difference of two perfect cubes \(x^3 - y^3 = (x - y)(x^2 + xy + y^2)\)
   c) Sum of two perfect cubes \(x^3 + y^3 = (x + y)(x^2 - xy + y^2)\)
   d) Perfect square trinomials \(x^2 + 2xy + y^2 = (x + y)^2\)
      \(x^2 - 2xy + y^2 = (x - y)^2\)

2) A general strategy for factoring:
   a) Factor out the greatest common factor
   b) For a binomial, look for:
      1. Difference of two perfect squares
      2. difference of two perfect cubes
      3. sum of two perfect cubes
   c) For a trinomial, look for:
      1. a perfect square trinomial
      2. try FOIL, grouping, or trial and error
   d) For 4 or more terms, use grouping
   e) Go back and factor each factor completely.
G. Examples

1) \[ 10ax^2 - 40ay^2 = 10a(x^2 - 4y^2) \]
\[ = 10a(x - 2y)(x + 2y) \]
Factoring out the GCF

Factoring the difference of two perfect squares

2) \[ 64x^6 - 1 = (8x^3 - 1)(8x^3 + 1) \]
\[ = (2x - 1)(4x^2 + 2x + 1)(2x + 1)(4x^2 - 2x + 1) \]
Difference of 2 perfect squares

Sum and difference of 2 perfect cubes

3) \[ 2x^2 - 20ax + 50a^2 = 2(x^2 - 10ax + 25a^2) \]
\[ = 2(x^2 - 2(5a)x + (5a)^2) \]
Factor out GCF

Rewrite to recognize a perfect square trinomial

Factor a perfect square trinomial

4) \[ 3x^2 - 10x - 8 \]
- This is not a perfect square trinomial, so try grouping.
- The lead coefficient is 3 and the constant is -8, their product is -24.
- The coefficient of the x term is -10.
- Find two numbers that multiply to be -24 and add up to be -10.
- The numbers are -12 and 2.
- Rewrite -10x as (-12x + 2x) then use grouping

\[ 3x^2 - 10x - 8 = 3x^2 - 12x + 2x - 8 \]
\[ = (3x^2 - 12x) + (2x - 8) \]
Rewriting with 4 terms to use grouping

Grouping the first two terms together and the last two terms together.

Factor out the GCF from each group

Factor out the GCF of (x-4)

5) \[ x^2 - 9y^2 + 12x + 36 = (x^2 + 12x + 36) - 9y^2 \]
\[ = (x + 6)^2 - (3y)^2 \]
Rearranging the terms to group as the difference of 2 perfect squares.

Rewriting as the difference of two perfect squares

Factoring the difference of two perfect squares
6) \[ x^3 - x^2y - xy^2 + y^3 = (x^3 - x^2y) - (xy^2 - y^3) \]
   \[ = x^2(x - y) - y^2(x - y) \] \textbf{Grouping}
   \[ = (x - y)(x^2 - y^2) \] \textbf{Factor GCF out of each group}
   \[ = (x - y)(x - y)(x + y) \] \textbf{Factor the difference of two squares}

**Problem Set #3:**

For problems 1 – 3, perform the indicated operations. For problems 4 – 15 factor the polynomial. Simplify completely whenever possible.

1) \((2x + 3)(x - 4) + 4x(x - 2)\)

2) \((2x^2 + x - 3)(-x^2 - 4x + 2)\)

3) \((x + 3y)^3\)

4) \(6x^3 + 21x^2 - 12x\)

5) \(x^6 - 8y^9\)

6) \(4x^2 - 25y^8\)

7) \(6x^2 - 5x - 6\)

8) \(4x^2 - 4xy - 4 + y^2\)

9) \(8x^3 + 12x^2 - 2x - 3\)

10) \(16x^4 - 8x^3y - 2xy^3 + y^4\)

11) \(x^3 + 64\)

12) \(27x^3 - 1\)

13) \(2x^4 - 162\)

14) \(6x^2 - 17x + 12\)

15) **Challenge:** \(x^2 - 12x + 36 - 49y^2\)
4. **Rational Expressions**

- A quotient of polynomials is called a rational expression.
- To simplify, add, subtract, multiply, or divide rational expressions, we use the same rules we use for fractions of real numbers.

A. **Examples**

**Simplifying Algebraic Fractions**

1) \[ \frac{x^2 + 2x + 1}{x^2 - 1} = \frac{(x + 1)(x + 1)}{(x + 1)(x - 1)} \]

   - Completely factor numerator and denominator
   - Factor out the common factors
   - Simplify

\[ = \frac{(x + 1)}{(x - 1)} \]

\[ = \frac{x + 1}{x - 1} \]

**Multiplying Algebraic Fractions**

2) \[ \frac{x^2 - 6x + 9}{x^2 - 1} \cdot \frac{2x - 2}{x - 3} = \frac{(x^2 - 6x + 9)(2x - 2)}{(x^2 - 1)(x - 3)} \]

   - Multiplication of fractions
   - Factor
   - Factor out common factors

\[ = \frac{(x - 3)(x - 3)(2)(x - 1)}{(x - 1)(x + 1)(x - 3)} \]

\[ = \frac{(x - 3)}{(x - 3)} \cdot \frac{(x - 1)}{(x - 1)} \cdot \frac{2(x - 3)}{(x + 1)} \]

\[ = \frac{2(x - 3)}{(x + 1)} \]

**Dividing Algebraic Fractions**

3) \[ \frac{x + 2}{2x - 3} \div \frac{x^2 - 4}{2x^2 - 3x} = \frac{x + 2}{2x - 3} \cdot \frac{2x^2 - 3x}{x^2 - 4} \]

   - Division of fractions

\[ = \frac{(x + 2)(2x - 3)}{(2x - 3)(x - 2)(x + 2)} \]

\[ = \frac{x}{x - 2} \]
**Adding/Subtracting Algebraic Fractions**

4) \[ \frac{6}{(3x-2)} + \frac{5}{x} - \frac{2}{x^2(3x-2)} \]

The LCD is \( x^2(3x-2) \)

\[ = \frac{6x^2}{x^2(3x-2)} + \frac{5x(3x-2)}{x^2(3x-2)} - \frac{2}{x^2(3x-2)} \]

Rewrite each fraction with the LCD

\[ = \frac{6x^2 + 5x(3x-2) - 2}{x^2(3x-2)} \]

Add and subtract the fractions

\[ = \frac{6x^2 + 15x^2 - 10x - 2}{x^2(3x-2)} \]

Simplify

\[ = \frac{21x^2 - 10x - 2}{x^2(3x-2)} \]

**Simplifying Complex Rational Expressions**

B. To simplify a complex rational expression, multiply the numerator and denominator by the LCD of all the included fractions.

1) Example:

\[ \frac{3}{2x-2} + \frac{1}{x+1} = \frac{3}{2(x-1)} \frac{1}{(x+1)} \]

Factor all polynomials

Note that the LCD of all four fractions is \( 2(x-1)(x+1) \)

\[ = \frac{3}{2(x-1)} \frac{1}{(x+1)} \frac{1}{(x-1)} \frac{x}{(x-1)(x+1)} \]

Multiply by the LCD

\[ = \frac{3}{2(x-1)} \cdot 2(x-1)(x+1) - \frac{1}{(x+1)} \cdot 2(x-1)(x+1) \]

Distribute

\[ = \frac{3}{2(x-1)} \cdot 2(x-1)(x+1) + \frac{x}{(x-1)(x+1)} \cdot 2(x-1)(x+1) \]

Simplify
Problem Set #4:

1) \[ \frac{6x^2yz^3 - xy^2z}{xyz} \]

2) \[ \frac{x^2 - 25}{x^3 - 125} \]

3) \[ \frac{2}{3x + 1} - \frac{9}{9x^2 + 6x + 1} \]

4) \[ \frac{\frac{a + b}{b}}{\frac{1}{a} - \frac{1}{b}} \]
5) \[
\frac{1}{x + h} - \frac{1}{h}
\]

6) \[
\frac{x-1}{3x^2-12}, \frac{x^2+2x-3}{x+3}
\]

7) \[
\frac{x^2+8}{x^2-1} \div \frac{x^2+5x+6}{x^2+2x-3}
\]

8) \[
\frac{3}{x-1} - \frac{4}{x-2}
\]

9) \[
\frac{x}{x^2-2x-24} = \frac{x}{x^2-7x+6}
\]

10) \[
\frac{4x^2 + x - 6}{x^2 + 3x + 2} - \frac{3x}{x + 1} + \frac{5}{x + 2}
\]

11) \[
\frac{x^2 + x - 12}{x^2 + x - 30} \div \frac{x^2 + 5x + 6}{x^2 - 2x - 3} \div \frac{x + 3}{x^2 + 7x + 6}
\]

12) \[
\frac{3}{x-2} - \frac{4}{x+2}
\]

13) \[
\frac{1}{x + 1} \div \frac{1}{x^2 - 2x - 3} + \frac{1}{x - 3}
\]
5. **Solving Equations**

A. To solve a linear equation, combine like terms and isolate the variable.

Solve for $x$: \[4x - 3 = -5x + 6\]

\[4x - 3 + 5x + 3 = -5x + 6 + 5x + 3\]  
\[9x = 9\]  
\[x = 1\]  

*Check to verify that $x = 1$ is a solution!*

B. To solve an equation with fractions, clear the fractions by multiplying everything by the LCD.

Solve for $x$: \[\frac{4}{x + 2} + \frac{2}{x - 2} = \frac{5x - 7}{x^2 - 4}\]  
The LCD is $x^2 - 4$

\[(x^2 - 4)\left(\frac{4}{x + 2}\right) + (x^2 - 4)\left(\frac{2}{x - 2}\right) = (x^2 - 4)\left(\frac{5x - 7}{x^2 - 4}\right)\]

\[(x - 2)(4) + (x + 2)(2) = (5x - 7)\]

\[4x - 8 + 2x + 4 = 5x - 7\]

\[6x - 4 = 5x - 7\]

\[x = -3\]

*Check to verify that $x = -3$ is a solution!*

C. To solve a polynomial equation, first try factoring and using the zero product property.

1) Solve for $x$: \[75x^3 + 35x^2 - 10x = 0\]

\[5x(15x^2 + 7x - 2) = 0\]

\[5x(5x - 1)(3x + 2) = 0\]

So by the zero product property we have:

\[5x = 0 \quad \text{or} \quad (5x - 1) = 0 \quad \text{or} \quad (3x + 2) = 0\]

\[x = 0 \quad \text{or} \quad x = \frac{1}{5} \quad \text{or} \quad x = -\frac{2}{3}\]

*Check to verify the solutions!*
3) Solve for $x$: \[ -2x^2 + 6x - 3 = 0 \]

This is not factorable, so we will use the quadratic formula.

If $a \neq 0$, the solutions of the equation $ax^2 + bx + c = 0$ are given by:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

For the equation above, $a = -2$, $b = 6$, and $c = -3$ so we get:

\[ x = \frac{-6 \pm \sqrt{6^2 - 4(-2)(-3)}}{2(-2)} = \frac{-6 \pm \sqrt{36 - 24}}{-4} = \frac{-6 \pm \sqrt{12}}{-4} = \frac{-6 \pm 2\sqrt{3}}{-4} = \frac{3 \pm \sqrt{3}}{2} \]

4) Solve for $x$: \[ (x - 3)^2 = 17 \]

Using the principal of square roots

\[ (x - 3)^2 = 17 \]

\[ \sqrt{(x - 3)^2} = \pm\sqrt{17} \]

\[ x - 3 = \pm\sqrt{17} \]

\[ x = 3 \pm \sqrt{17} \]

Using the quadratic formula

\[ (x - 3)^2 = 17 \]

\[ x^2 - 6x + 9 = 17 \]

\[ x^2 - 6x - 8 = 0 \]

\[ x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-8)}}{2(1)} \]

\[ x = \frac{6 \pm \sqrt{36 + 32}}{2} \]

\[ x = \frac{6 \pm \sqrt{68}}{2} \]

\[ x = \frac{6 \pm 2\sqrt{17}}{2} \]

\[ x = 3 \pm \sqrt{17} \]
D) Solving an Equation Involving Absolute Value

Solve: \(3|2x - 3| - 8 = 25\)

\[
3|2x - 3| - 8 = 25 \quad \text{We need to isolate } |2x - 3|
\]

\[
3|2x - 3| = 33 \quad \text{Add 8 to both sides.}
\]

\[
|2x - 3| = 11 \quad \text{Divide both sides by 3.}
\]

\[
2x - 3 = 11 \quad \text{or} \quad 2x - 3 = -11 \quad \text{Rewrite the equation without absolute value bars.}
\]

\[
2x = 14 \quad 2x = -8 \quad \text{Add 3 to both sides of each equation.}
\]

\[
x = 7 \quad x = -4 \quad \text{Divide both sides of each equation by 2.}
\]

Check to verify the solutions!

The solution set is \{-4, 7\}.

Problem Set #5:

Solve the following equations. Check all your answers!

1) \((x + 5)^2 + 3 = (x - 2)^2\)

2) \[\frac{2}{2x + 5} + \frac{3}{2x - 5} = \frac{10x + 5}{4x^2 - 25}\]

3) \(48x^2 + 12x - 90 = 0\)

4) \[\frac{3x}{x - 2} + \frac{1}{x + 2} = \frac{-4}{x^2 - 4}\]

5) \(16x^2 = 49\)

6) \((x + 4)^2 = 31\)

7) \(x^2 - 6x - 3 = 0\)

8) \[\frac{3}{x + 3} = \frac{5}{2x + 6} + \frac{1}{x - 2}\]
9) \[ \frac{6}{x+3} - \frac{5}{x-2} = \frac{-20}{x^2 + x - 6} \]

10) \[ 2|2x - 1| - 5 = 9 \]

11) \[ -3|3x - 13| + 7 = -8 \]

12) \[ 2x^2 = 3 - 4x \]
### 6. Answers:

<table>
<thead>
<tr>
<th>Problem Set #1</th>
<th>Problem Set #2</th>
<th>Problem Set #3</th>
<th>Problem Set #4</th>
<th>Problem Set #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \left[ \frac{11}{2}, \infty \right) )</td>
<td>1. (-12x^2)</td>
<td>1. (6x^2 - 13x - 12)</td>
<td>1. (6xz^2 - y)</td>
<td>1. (x = -\frac{12}{7})</td>
</tr>
<tr>
<td>2. ([1,7])</td>
<td>2. (-\frac{27a^6}{b^{15}})</td>
<td>2. (-2x^4 - 9x^3 + 3x^2 + 14x - 6)</td>
<td>2. (\frac{x + 5}{x^2 + 5x + 25})</td>
<td>2. All real #s</td>
</tr>
<tr>
<td>3. ((-1,2])</td>
<td>3. (\frac{12}{y^5})</td>
<td>3. (x^3 + 9x^2y + 27xy^2 + 27y^3)</td>
<td>3. (\frac{6x - 7}{(3x + 1)(3x + 1)})</td>
<td>3. (x = \frac{5}{4}, -\frac{3}{2})</td>
</tr>
<tr>
<td>4. (\left[ -\frac{19}{6}, \infty \right) )</td>
<td>4. (\frac{3}{x^3y^2})</td>
<td>4. (-15. Answers will be provided when classes begin.)</td>
<td>4. (\frac{a^2 + b^2}{b - a})</td>
<td>4. (x = -\frac{1}{3})</td>
</tr>
<tr>
<td>5. ((-1,7))</td>
<td>5. 1</td>
<td></td>
<td>5. (-\frac{1}{x(x+h)})</td>
<td>5. (x = \pm \frac{7}{4})</td>
</tr>
<tr>
<td>6. ((-\infty, \frac{1}{3}) ) or ((5, \infty))</td>
<td>6. (\frac{2a^2}{b})</td>
<td>6. (\frac{(x - 1)^2}{3x^2 - 12})</td>
<td>6. (x = -4 \pm \sqrt{31})</td>
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<tr>
<td>7. ((-6,0))</td>
<td>7. (\sqrt{\frac{6xy}{2y^2}})</td>
<td>7. (\frac{x^2 - 2x + 4}{x^2 + x + 1})</td>
<td>7. (x = 3 \pm 2\sqrt{3})</td>
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<tr>
<td>8. ((-\infty, -8) ) or ((16, \infty))</td>
<td>8. (\frac{xy\sqrt{6y}}{3})</td>
<td>8. (\frac{-x^2 - 3x - 2}{x^2 + 12x - 13})</td>
<td>8. (x = -8)</td>
<td></td>
</tr>
<tr>
<td>9. ((-\infty, -1) ) or ([3, \infty))</td>
<td>9. (6xz^2\sqrt{2x^2y^2})</td>
<td>9. (-\frac{5x}{(x - 1)(x + 4)(x - 6)})</td>
<td>9. (x = 7)</td>
<td></td>
</tr>
<tr>
<td>10. ([2,6])</td>
<td>10. (\frac{-2\sqrt{9x}}{3x})</td>
<td>10. (\frac{x - 1}{x + 2})</td>
<td>10. (x = -3, 4)</td>
<td></td>
</tr>
<tr>
<td>11. (7\sqrt{5} + 14)</td>
<td>11. (\frac{(x + 4)(x + 2)}{x - 5})</td>
<td>11. (x = \frac{8}{3}, 6)</td>
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</tr>
<tr>
<td>12. (\frac{11\sqrt{7} + 11\sqrt{3}}{4})</td>
<td>12. (-\frac{x + 14}{7})</td>
<td>12. (x = \frac{-2 \pm \sqrt{10}}{2})</td>
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<tr>
<td>13. (\frac{x - 3}{x + 2})</td>
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